



Problem:

Find the general solution of the first order differential equation:

$$y' \sin^2 5x - y^2 + 5 = 0.$$

Solution:

This is an equation with separable variables

$$\Rightarrow \frac{dy}{y^2 + 5} = \frac{dx}{\sin^2 5x}, \text{ let's integrate, } \int \frac{dy}{y^2 + 5} = \int \frac{dx}{\sin^2 5x}, \frac{1}{\sqrt{5}} \tan^{-1} \left(\frac{y}{\sqrt{5}} \right) = -\frac{1}{5} \cdot \cot 5x + C, \text{ where } C \text{ is}$$

the arbitrary constant \Rightarrow we obtain the general solution to the initial equation:

$$\boxed{y = \sqrt{5} \cdot \tan \left(-\frac{1}{\sqrt{5}} \cot 5x + C \right)} \quad (\text{has been renamed from } \sqrt{5}C \text{ to } C).$$