



Problem:

Find the general integral of the differential equation:

$$x\sqrt{4+y^2}dx + y\sqrt{1+x^2}dy = 0.$$

Solution:

This is an equation with separable variables:

$$\frac{ydy}{\sqrt{4+y^2}} = -\frac{xdx}{\sqrt{1+x^2}}, \Rightarrow \text{let's integrate, } \int \frac{ydy}{\sqrt{4+y^2}} = -\int \frac{xdx}{\sqrt{1+x^2}}, \quad \frac{1}{2} \int \frac{dy^2}{\sqrt{4+y^2}} = -\frac{1}{2} \int \frac{dx^2}{\sqrt{1+x^2}},$$

$\sqrt{4+y^2} = -\sqrt{1+x^2} + C$, where C is the arbitrary constant \Rightarrow we have obtained the desired general integral of the equation:

$$\sqrt{4+y^2} + \sqrt{1+x^2} = C.$$

$$\text{Answer: } \sqrt{4+y^2} + \sqrt{1+x^2} = C.$$