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Problem:

Find the general solution of the differential equation:

$(3+e^x)yy' = e^x$.

Solution:

This is a 1st order non-linear equation with separable variables, since

$y' = \frac{dy}{dx} \Rightarrow (3+e^x)\cdot y\frac{dy}{dx} = e^x \Rightarrow y\cdot dy = \frac{e^x}{e^x+3}dx$, let's integrate $\Rightarrow \int ydy = \int \frac{e^x}{e^x+3}dx \Rightarrow$

$\frac{y^2}{2} = \int \frac{e^x}{e^x+3}dx = \int \frac{de^x}{e^x+3} = \boxed{e^x = t \Rightarrow} = \int \frac{dt}{t+3} = \ln|t+3| + C_0 = \ln(e^x+3) + C_0 \Rightarrow$

$y^2 = 2\ln(e^x+3) + C$, where C is the arbitrary constant $\Rightarrow$ we obtain the general solutions of the equation: (there are two of them):

$y = \sqrt{2\ln(e^x+3)+C}, \qquad y = -\sqrt{2\ln(e^x+3)+C}$.