Problem:

Find the general solution of the higher order differential equation:

$y^{(4)} = \cos 5x.$

Solution:

$$y^{(4)} = \cos 5x \,, \Rightarrow\ y^{(3)} = \int \cos 5x\, dx = \frac{1}{5}\sin 5x + C_1,$$

$$y^{(2)} = \int \left(\frac{1}{5}\sin 5x + C_1\right) dx = -\frac{1}{25}\cos 5x + C_1 x + C_2,$$

$$y' = \int \left(-\frac{1}{25}\cos 5x + C_1 x + C_2\right) dx = -\frac{1}{125}\sin 5x + \frac{C_1}{2}x^2 + C_2 x + C_3,$$

$$y = \int \left(-\frac{1}{125}\sin 5x + \frac{C_1}{2}x^2 + C_2 x + C_3\right) dx = \frac{1}{625}\cos 5x + \frac{C_1}{6}x^3 + \frac{C_2}{2}x^2 + C_3 x + C_4 \Rightarrow$$

The general solution of the initial equation will be:

$y(x) = \dfrac{\cos 5x}{625} + C_1 x^3 + C_2 x^2 + C_3 x + C_4$, where C_1, C_2, C_3, C_4 are arbitrary constants.