Problem:

Determine the type of the second order equation:

$u_{xx} + 2u_{xy} - 2u_{yz} + 2u_{yy} + 2u_{zz} = 0.$

Solution:

Let's determine the type of the equation: for it let's write the corresponding quadratic form and bring it to canonical form, using the Lagrange method (full square separation method):

$P = p_1^2 + 2p_1p_2 - 2p_2p_3 + 2p_2^2 + 2p_3^2 = (p_1 + p_2)^2 + p_2^2 - 2p_2p_3 + p_3^2 + p_3^2 =$

$= (p_1 + p_2)^2 + (p_2 - p_3)^2 + p_3^2 = \xi_1^2 + \xi_2^2 + \xi_3^2,$ where $\xi_1 = p_1 + p_2, \xi_2 = p_2 - p_3, \xi_3 = p_3.$

We see that in the obtained canonical form all coefficients have the same sign (positive) ⇒ the initial equation is of elliptic type.

Answer: the equation is of elliptic type.