Problem:

At what values of λ the following quadratic form is positive definite:

$$f = 5x_1^2 + x_2^2 + \lambda x_3^2 + 4x_1x_2 - 2x_1x_3 - 2x_2x_3.$$

Solution:

Let's compose the matrix of this quadratic form:

$$A = \begin{pmatrix} 5 & 2 & -1 \\ 2 & 1 & -1 \\ -1 & -1 & \lambda \end{pmatrix}$$

According to the Sylvester criterion, the quadratic form $f(x_1, x_2, x_3)$ is positive definite if and only if all principal minors of its matrix A are positive $\Rightarrow$ we obtain:

$$\Delta_1 = |5| = 5 > 0, \qquad \Delta_2 = \begin{vmatrix} 5 & 2 \\ 2 & 1 \end{vmatrix} = 1 > 0, \qquad \Delta_3 = \begin{vmatrix} 5 & 2 & -1 \\ 2 & 1 & -1 \\ -1 & -1 & \lambda \end{vmatrix} = \lambda - 2 > 0 \Rightarrow \lambda > 2.$$

Answer: $\lambda \in (2, +\infty)$.