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Problem:

Find the coordinates of the vector X in the basis $e' = \{e_1'; e_2'; e_3'\}$, if $X = (1; -4; 8)$ in the basis $e = \{e_1; e_2; e_3\}$ and

$$\begin{cases} e_1' = e_1 + e_2 - 3e_3 \\ e_2' = \frac{3}{4}e_1 - e_2 \\ e_3' = -e_1 + e_2 + e_3 \end{cases}$$

Solution:

In the basis $e = \{e_1; e_2; e_3\}$ the coordinates of the vector $X = (1; -4; 8)$.

$$\begin{cases} e_1' = e_1 + e_2 - 3e_3 \\ e_2' = \frac{3}{4}e_1 - e_2 \\ e_3' = -e_1 + e_2 + e_3 \end{cases}$$

Let's write the transition matrix from $e\ to\ e'$:

$$A = \begin{pmatrix} 1 & \frac{3}{4} & -1 \\ 1 & -1 & 1 \\ -3 & 0 & 1 \end{pmatrix}$$

⇒ the formula for the coordinates of X in the basis e' is:

$$X = A \cdot X' \Rightarrow X' = A^{-1} \cdot X, \qquad A^{-1} = \frac{1}{\det A} \cdot \begin{pmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{pmatrix}, \qquad detA = \begin{vmatrix} 1 & \frac{3}{4} & -1 \\ 1 & -1 & 1 \\ -3 & 0 & 1 \end{vmatrix} = -1.$$

Computing the algebraic complements of the matrix, we obtain the inverse matrix:

$$A^{-1} = \begin{pmatrix} 1 & \frac{3}{4} & \frac{1}{4} \\ 4 & 2 & 2 \\ 3 & \frac{9}{4} & \frac{7}{4} \end{pmatrix} \Rightarrow X' = A^{-1} \cdot X = \begin{pmatrix} 1 & \frac{3}{4} & \frac{1}{4} \\ 4 & 2 & 2 \\ 3 & \frac{9}{4} & \frac{7}{4} \end{pmatrix} \cdot \begin{pmatrix} 1 \\ -4 \\ 8 \end{pmatrix} = \begin{pmatrix} 0 \\ 12 \\ 8 \end{pmatrix}$$

⇒ the desired coordinates of X in the basis e' will be:

$X' = (0; 12; 8)$.