Problem:

Determine the type of the system of linear partial differential equations of the first order:

$$\begin{cases} 3u_x + 4u_y - v_y + v_x = 0 \\ u_x + v_y - v_x = 0 \end{cases}$$

Solution:

The general form of the system is as follows:

$$\begin{cases} a_{11}u_x + a_{12}u_y + b_{11}v_x + b_{12}v_y = 0 \\ a_{21}u_x + a_{22}u_y + b_{21}v_x + b_{22}v_y = 0 \end{cases}$$

where

$a_{11} = 3, \quad a_{12} = 4, \quad a_{21} = 1, \quad a_{22} = 0,$

$b_{11} = 1, \quad b_{12} = -1, \quad b_{21} = -1, \quad b_{22} = 0.$

The corresponding characteristic form for our system will be:

$$Q(\lambda_1, \lambda_2) = \begin{vmatrix} a_{11}\lambda_1 + a_{12}\lambda_2 & b_{11}\lambda_1 + b_{12}\lambda_2 \\ a_{21}\lambda_1 + a_{22}\lambda_2 & b_{21}\lambda_1 + b_{22}\lambda_2 \end{vmatrix} = \begin{vmatrix} 3\lambda_1 + 4\lambda_2 & \lambda_1 - \lambda_2 \\ \lambda_1 & -\lambda_1 \end{vmatrix} =$$

$$= -3\lambda_1^2 - 4\lambda_1\lambda_2 - \lambda_1^2 + \lambda_1\lambda_2 = -4\lambda_1^2 - 3\lambda_1\lambda_2 = -(2\lambda_1)^2 - 2 \cdot 2\lambda_1 \cdot \frac{3}{4}\lambda_2 - \left(\frac{3}{4}\lambda_2\right)^2 + \left(\frac{3}{4}\lambda_2\right)^2 =$$

$$= -\left(2\lambda_1 + \frac{3}{4}\lambda_2\right)^2 + \left(\frac{3}{4}\lambda_2\right)^2 = -\xi_1^2 + \xi_2^2, \text{где } \xi_1 = 2\lambda_1 + \frac{3}{4}\lambda_2, \xi_2 = \frac{3}{4}\lambda_2.$$

We have brought the quadratic form to a normal form. We see that in the normal form of Q there are two coefficients of the opposite sign $\Rightarrow$ the initial system is of hyperbolic type.

Answer: hyperbolic type.