Problem:

Find out whether the set of integers divisible by 5, form a group with respect to the addition.

$M = \{n \in \mathbb{Z} \mid n \vdots 5\} = \{5k \mid k \in \mathbb{Z}\}$.

Solution:

Let's find out if M forms a group with respect to the addition. Let's check the axioms of the group:

a) Associativity:

$\forall\ 5n;\ 5m;\ 5k\ \in M\ (n; m; k\ \in \mathbb{Z}) \Rightarrow (5n + 5m) + 5k = 5n + 5m + 5k = 5n + (5m + 5k) = 5(n + m + k)$

$\Rightarrow$ associativity holds.

b) $n = 0 \in \mathbb{Z}\ \Rightarrow 5n = 5 \cdot 0 = 0 \in M;\ \forall\ 5m \in M \Rightarrow 5m + 5 \cdot 0 = 5 \cdot 0 + 5m = 5m \Rightarrow 5 \cdot 0 = 0$ is a neutral element.

c) $\forall n \in \mathbb{Z} \Rightarrow (-n) \in \mathbb{Z} \Rightarrow 5n + 5 \cdot (-n) = 5 \cdot (-n) + 5n = 0$ for any $5n \in M$ there exists an inverse element $5 \cdot (-n) \in M$.

All the axioms of the group hold $\Rightarrow M$ is a group with respect to the addition.