Problem:

Find out whether the set of powers of number 7 with integer exponents form a group with respect to multiplication.

$M = \{7^n \mid n \in \mathbb{Z}\}$.

Solution:

Let's find out whether set M forms a group with respect to multiplication. For that purpose let us check the axioms of the group:

a) associativity:

$\forall\, 7^n;\ 7^m;\ 7^k\ \in M\ (n; m; k\ \in \mathbb{Z}) \Rightarrow (\,7^n \cdot 7^m) \cdot 7^k\ =\ 7^n \cdot 7^m \cdot 7^k\ =\ 7^n \cdot \left(7^m \cdot 7^k\,\right) = 7^{m+n+k} \Rightarrow$

Associativity holds.

b) When $n = 0\ \Rightarrow 7^0 = 1;\ \forall\, 7^m \in M \Rightarrow 7^m \cdot 7^0 =\ 7^m \cdot 1 = 7^0 \cdot\ 7^m = 7^m \Rightarrow 7^0 = 1 \in M$ is a neutral element.

c) $\forall n \in \mathbb{Z} \Rightarrow (-n) \in \mathbb{Z};\ \ 7^n \cdot 7^{-n} = 7^{-n} \cdot 7^n = 7^0 = 1 \Rightarrow$ for any $7^n \in M$ there is an inverse element

$7^{-n} \in M$.

All axioms of the group are satisfied $\Rightarrow M$ is a group with respect to multiplication.