Problem:

Prove the statement by mathematical induction: $(n^5 - n)$ is a multiple of 5 for all natural n.

Solution:

Using the method of mathematical induction let's prove that $(n^5 - n) \vdots 5$, for any $n \in \mathbb{N}$.

($a \vdots b$ –means a is divided by b).

a) Let's check the validity of the statement when $n = 1$.

$1^5 - 1 = 0 \vdots 5 \Rightarrow$ the statement is true.

b) Let the statement be true when $n = k$. Let's prove its validity when $n = k + 1$.

We have $(k^5 - k) \vdots 5, \Rightarrow (k+1)^5 - (k+1) = k^5 + 5k^4 + 10k^3 + 10k^2 + 5k + 1 - k - 1 = k^5 - k +$

$+5(k^4 + 2k^3 + 2k^2 + k) \vdots 5$, since $(k^5 - k) \vdots 5 \Rightarrow \left((k+1)^5 - (k+1)\right) \vdots 5 \Rightarrow$ the statement $(n^5 - n) \vdots 5$ is true for every $n \in \mathbb{N}$.