



Problem:

Find the sequence $\{a_n\}$, that satisfies the recurrence relation $2 \cdot a_{n+2} + 10 \cdot a_{n+1} + 8 \cdot a_n = 0$ and the initial conditions $a_1 = 3, a_2 = 9$.

Solution:

$$2 \cdot a_{n+2} + 10 \cdot a_{n+1} + 8 \cdot a_n = 0, a_1 = 3, a_2 = 9.$$

Consider the characteristic equation of the recurrence relation:

$P(\lambda) = 2\lambda^2 + 10\lambda + 8 = 0 \Rightarrow \lambda^2 + 5\lambda + 4 = 0, \lambda_1 = -4, \lambda_2 = -1 \Rightarrow$ the general solution of the relation has the form $a_n = C_1\lambda_1^n + C_2\lambda_2^n$, i.e., $a_n = C_1 \cdot (-4)^n + C_2 \cdot (-1)^n$. Using the initial conditions, we obtain:

$$\begin{cases} \alpha_1 = -4C_1 - C_2 = 3 \Rightarrow C_2 = -4C_1 - 3 \\ \alpha_2 = 16C_1 + C_2 = 9 \end{cases} \Rightarrow 16C_1 - 4C_1 - 3 = 9 \Rightarrow C_1 = 1 \Rightarrow C_2 = -4 - 3 = -7 \Rightarrow$$

$$\Rightarrow a_n = -4n - 7 \cdot (-1)^n.$$

$$\text{Answer: } a_n = (-4)^n - 7 \cdot (-1)^n.$$