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Problem:

Find the general integral of the differential equation:

$x\sqrt{4+y^2}dx + y\sqrt{1+x^2}dy = 0.$

Solution:

This is an equation with separable variables:

$$\frac{ydy}{\sqrt{4+y^2}} = -\frac{xdx}{\sqrt{1+x^2}}, \Rightarrow \text{let's integrate,} \int \frac{ydy}{\sqrt{4+y^2}} = -\int \frac{xdx}{\sqrt{1+x^2}},$$

$$\frac{1}{2}\int \frac{dy^2}{\sqrt{4+y^2}} = -\frac{1}{2}\int \frac{dx^2}{\sqrt{1+x^2}}, \qquad \sqrt{4+y^2} = -\sqrt{1+x^2} + C,$$ where C is the arbitrary constant.

We have obtained the general integral of the equation: $\sqrt{4+y^2} + \sqrt{1+x^2} = C$, or explicitly we have:

$y^2 = \left(C - \sqrt{1+x^2}\right)^2 - 4.$

Answer: $\sqrt{4+y^2} + \sqrt{1+x^2} = C.$