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Problem:

Find a solution to Cauchy problem:

$$y' + \frac{y}{x} = 3x, \qquad y(1) = 1.$$

Solution:

First let's solve the homogeneous equation:

$$y' + \frac{y}{x} = 0 \Rightarrow \frac{dy}{y} = -\frac{dx}{x}, \qquad \int \frac{dy}{y} = -\int \frac{dx}{x}, \qquad \ln|y| = -\ln|x| + C_1, \qquad |y| = |x|^{-1} \cdot e^{C_1} \Rightarrow y = \frac{C}{x},$$

where C is the arbitrary constant.

According to the method of variation of the arbitrary constant we obtain:

$$C = C(x), \qquad y' = \frac{C'x - C}{x^2} \Rightarrow \frac{C'x - C}{x^2} + \frac{C}{x^2} = 3x \Rightarrow C' = 3 \Rightarrow C(x) = 3x + C_1, \Rightarrow$$

$$y = \frac{3x + C_1}{x}, \qquad y(1) = 1 \Rightarrow \frac{3 + C_1}{1} = 1 \Rightarrow C_1 = -2 \Rightarrow y = 3 - \frac{2}{x},$$ will be the desired solution.

Answer: $y = 3 - \frac{2}{x}$.