Problem:

Investigate linear dependence for the following system of functions: e^x , xe^x , x^2e^x .

Solution:

$$y_1 = e^x, y_2 = xe^x, y_3 = x^2e^x.$$

Using the definition of linear dependence $\alpha_1 y_1 + \alpha_2 y_2 + \alpha_3 y_3 = 0$, $\Rightarrow e^x(\alpha_1 + \alpha_2 x + \alpha_3 x^2) \equiv 0$, $\Rightarrow \alpha_1 = \alpha_2 = \alpha_3 = 0 \Rightarrow$ the given functions are linearly independent.

Answer: linearly independent.