Problem:

Write the differential equation of the hyperbola $x^2 - y^2 = 2ax$.

Solution:

Let's differentiate both parts of the equation with respect to $x \Rightarrow 2x - 2y \cdot y' = 2a \Rightarrow$ let's differentiate again with respect to x (both parts) $\Rightarrow 2 - 2(y' \cdot y' + y \cdot y'') = 0, \Rightarrow$ the desired differential equation will be

$y \cdot y'' + (y')^2 = 1$. (we need to differentiate the initial equation with respect to x until we get rid of all constants).

Answer: $y \cdot y'' + (y')^2 = 1$.