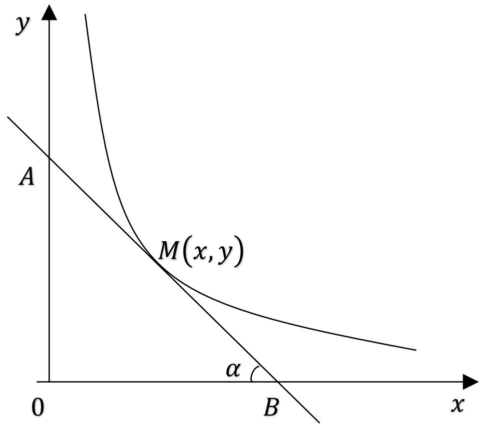
Problem:

Write a differential equation of the family of curves, in which the segment of any tangent, closed between the coordinate axes, is divided by the tangent point $M(x,y)$ in relation to $|AM|:|MB| = 2:1$, where A is the point of intersection of the tangent with the Oy axis, B is the point of intersection with the Ox axis.

Solution:



The equation of the tangent at the point $M(x,y)$ will be $Y - y = = y'(X - x)$ (since the angular coefficient is $k = \tan\alpha = y'(x)$), where (X,Y) is the current point of the tangent. When $X_A = 0 \Rightarrow Y_A = -y'x + y$, $A(0; y - y'x)$, when $Y_B = 0 \Rightarrow X_B = x - \frac{y}{y'}$, $\Rightarrow B\left(x - \frac{y}{y'}, 0\right)$, M divides AB in relation to $2:1, \Rightarrow x = \frac{X_A + 2X_B}{3}$, from both equalities imply:

$$x = \frac{2}{3}\left(x - \frac{y}{y'}\right) \Rightarrow xy' + 2y = 0, \Rightarrow y' = -\frac{2y}{x}.$$

This is the differential equation of the desired curves.

Answer: $y' = -\frac{2y}{x}$.