



Problem:

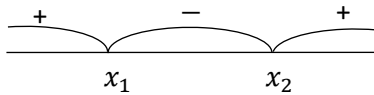
In case of what a the equation $-10a - 10|a - x| - 85 = -15x - x^2$ has at least one root in an interval $(3; 7)$?

Solution:

$-10a - 10|a - x| - 85 = -15x - x^2$, the equation must have at least one root in an interval $(3; 7)$.

$\Rightarrow -10(a - x) - 10|a - x| = 85 - 15x - x^2 + 10x \Rightarrow x^2 + 5x - 85 = 10(|a - x| + a - x)$, now since $|t| \geq -t \forall t \in (-\infty; +\infty) \Rightarrow$ for all a and x we have: $|a - x| \geq -(a - x) \Rightarrow |a - x| + a - x \geq 0, \forall a, x \in (-\infty; +\infty)$, therefore $x^2 + 5x - 85 \geq 0$, and since according to the condition the initial equation must have at least one root from $(3; 7) \Rightarrow$ for that root $x_0 \Rightarrow x_0^2 + 5x_0 - 85 \geq 0$, but we see, that the solution to the inequality $x^2 + 5x - 85 \geq 0$ (*) will be: $(-\infty; x_1) \cup (x_2; +\infty)$, where

$$x_1 = \frac{-5 - \sqrt{365}}{2}, \quad x_2 = \frac{-5 + \sqrt{365}}{2}, \quad \text{but } \frac{-5 + \sqrt{365}}{2} > 7 \quad (365 > 361), \quad \frac{-5 - \sqrt{365}}{2} < 3 \Rightarrow$$



No number from $(3; 7)$ satisfies the inequality (*) \Rightarrow such a doesn't exist.

Answer: such a doesn't exist.