Problem:

In case of what values of the parameter a the equation $-4a + 4|a - x| + 51 = 11x - x^2$ has at least one solution in an interval $(6; 9)$?

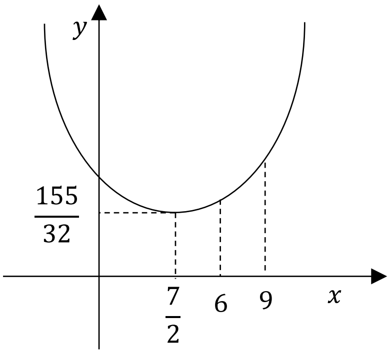
Solution:

When $a \geq x \Rightarrow -4a + 4(a - x) + 51 = 11x - x^2 \Rightarrow\ x^2 - 15x + 51 = 0 \Rightarrow$

$$x_1 = \frac{15 + \sqrt{21}}{2} > 9\ (21 > 9), \qquad x_2 = \frac{15 - \sqrt{21}}{2} < 6\ (9 < 21) \Rightarrow$$

when $x \leq a$ the equation has no solutions on $(6; 9) \Rightarrow\ x > a \Rightarrow\ -4a + 4(x - a) + 51 = 11x - x^2 \Rightarrow$

$$a = \frac{x^2 - 7x + 51}{8}\ , the\ graph\ y = \frac{x^2 - 7x + 51}{8}\ is\ a\ parabola: y = \frac{1}{8}\left(x - \frac{7}{2}\right)^2 + \frac{155}{32} \Rightarrow$$



the initial equation will have at least one root on $(6; 9)$ only if

$$a > \frac{6^2 - 7 \cdot 6 + 51}{8} = \frac{45}{8}\ and\ a < \frac{9^2 - 7 \cdot 9 + 51}{8} = \frac{69}{8} \Rightarrow \frac{45}{8} < a < \frac{69}{8}.$$

$$Answer: a \in \left(\frac{45}{8}; \frac{69}{8}\right).$$