Problem:

Determine the character of the resting point of the following system:

$$x' = -2x + \frac{1}{3}y, \qquad y' = -2x + \frac{1}{2}y.$$

Solution:

$$\begin{cases} x' = -2x + \frac{1}{3}y & M(x;y) = -2x + \frac{1}{3}y \\ y' = -2x + \frac{1}{2}y & N(x;y) = -2x + \frac{1}{2}y \end{cases} \Rightarrow \text{ the resting point } x = y = 0,\ M(0;0) = N(0;0) = 0.$$

Let's find the eigenvalues of the matrix of the system:

$$A = \begin{pmatrix} -2 & \frac{1}{3} \\ -2 & \frac{1}{2} \end{pmatrix}, \qquad \det(A - \lambda I) = \begin{vmatrix} -2-\lambda & \frac{1}{3} \\ -2 & \frac{1}{2} - \lambda \end{vmatrix} = 0, \qquad (2+\lambda)\left(\frac{1}{2} - \lambda\right) - \frac{2}{3} = 0, \lambda^2 + \frac{3}{2}\lambda - \frac{1}{3} = 0,$$

$6\lambda^2 + 9\lambda - 2 = 0, \lambda_1 = \frac{-9+\sqrt{129}}{2}, \lambda_2 = \frac{-9-\sqrt{129}}{2}, \lambda_1 \cdot \lambda_2 < 0 \Rightarrow$ the singular point will be the saddle.

Next $\lambda_1 > 0 \Rightarrow$ according to Lyapunov's first theorem, the resting point of the system is unstable.

Answer: the resting point of the system is unstable.