Problem:

Determine the character of the resting point of the following system:

$x' = -y, \quad y' = x - 2y.$

Solution:

$(*)\begin{cases} x' = -y \\ y' = -x - y \end{cases}, \quad \begin{matrix} M(x;y) = -y, \\ N(x;y) = x + 2y \end{matrix} \quad \Rightarrow$ the resting point $x = y = 0, \; M(0;0) = N(0;0) = 0.$

Let's determine the eigenvalues of the matrix of the system.

$A = \begin{pmatrix} 0 & -1 \\ 1 & -2 \end{pmatrix}. \det(A - \lambda I) = \begin{vmatrix} -\lambda & -1 \\ 1 & -2-\lambda \end{vmatrix} = 0 \Rightarrow \lambda(\lambda + 2) + 1 = 0, \lambda^2 + 2\lambda + 1 = 0, \lambda_1 = \lambda_2 = -1.$

$Re(\lambda_1) = Re(\lambda_2) = -1 < 0 \Rightarrow$ according to Lyapunov's first theorem, the system $(*)$ is unstable at the resting point.

Answer: the resting point of the system is unstable.