Problem:

Reduce the equation of the second-order curve $f(x,y)=0$ to canonical form and find its points of intersection with the straight line $Ax+By+C=0$.

$2x^2+4x+y^2-2=0, \qquad 2x+y-2=0.$

Solution:

Let's reduce the equation of the curve to canonical form:

$$2(x^2+2x+1)+y^2-4=0, \qquad \boxed{\frac{(x+1)^2}{2}+\frac{y^2}{4}=1}.$$

It's an ellipse.

Let's find the points of intersection of the ellipse and the given straight line:

$$\begin{cases}2x+y+2=0\\2x^2+4x+y^2-2=0\end{cases} \Rightarrow y=-2x-2 \Rightarrow 2x^2+4x+(-2x-2)^2-2=0,$$

$$3x^2+6x+1=0 \Rightarrow x=\frac{-3+\sqrt{6}}{3} \Rightarrow y=-2x-2=\frac{-2\sqrt{6}}{3}, \qquad x=\frac{-3-\sqrt{6}}{3} \Rightarrow y=\frac{2\sqrt{6}}{3} \Rightarrow$$

the points of intersection will be:

$$\left(\frac{-3+\sqrt{6}}{3};-\frac{2\sqrt{6}}{3}\right), \qquad \left(\frac{-3-\sqrt{6}}{3};\frac{2\sqrt{6}}{3}\right).$$