



Problem:

Prove the identity:  $\overline{B \setminus A} = \bar{B} \cup A$ .

Solution:

$\bar{A}$  is a complement to the set  $A$ ,  $U$  is a universal set, i.e.,  $\bar{A} = U \setminus A$ .

Let's prove that  $A \setminus B = A \cap \bar{B}$ .  $A \setminus B = \{x \in A, x \notin B\}$ .

For any  $x \in A \setminus B \Rightarrow x \in A, x \notin B \Rightarrow x \in U \setminus B = \bar{B} \Rightarrow x \in A, x \in \bar{B} \Rightarrow x \in A \cap \bar{B}$ .

Similarly,  $\forall x \in A \cap \bar{B} \Rightarrow x \in A, x \in \bar{B} \Rightarrow x \in U \setminus B \Rightarrow x \notin B \Rightarrow x \in A, x \notin B \Rightarrow x \in A \setminus B, \Rightarrow$

$\Rightarrow x \in A \setminus B \Leftrightarrow x \in A \cap \bar{B} \Rightarrow A \setminus B = A \cap \bar{B} (*)$ .

Now according to De Morgan's law  $\Rightarrow \overline{B \setminus A} = \boxed{\text{from } (*) \Rightarrow} = \overline{B \cap \bar{A}} = \bar{B} \cup \bar{\bar{A}} = \boxed{\bar{\bar{A}} = A \text{ is a double complement}} \Rightarrow = \bar{B} \cup A$ .