



Problem:

Prove equalities using the properties of set operations and definitions of operations.

$$C \subseteq D \Rightarrow A \times C \subseteq B \times D.$$

Solution:

$$A \times C = \{(a, c) \mid a \in A, c \in C\}$$

$$B \times D = \{(b, d) \mid b \in B, d \in D\}$$

Only  $C \subseteq D$  is known, from  $C \subseteq D$  does not imply  $A \times C \subseteq B \times D$ , that is, the equality will be true if  $A \subseteq B$ .

Let us prove with an example that the statement  $C \subseteq D \Rightarrow A \times C \subseteq B \times D$  is false.

Let sets  $C = \{1, 2\}$ ,  $D = \{1, 2, 3, 4\}$  be given, where  $C \subseteq D$

$A = \{1, 3, 5, 6\}$ ,  $B = \{3, 7\}$ , where  $A \not\subseteq B$

$$A \times C = \{(1, 1), (1, 3), (1, 5), (1, 6), (2, 1), (2, 3), (2, 5), (2, 6)\}$$

$$B \times D = \{(1, 3), (1, 7), (2, 3), (2, 7), (3, 3), (3, 7), (4, 3), (4, 7)\}$$

Thus, from the example it is clear that  $(1, 1) \notin B \times D \Rightarrow A \times C \not\subseteq B \times D$  under the condition that  $C \subseteq D$ .