Problem:

Find residues:

$\operatorname{Res}_{-3} \dfrac{z+2}{z(z+3)^2}$.

Solution:

$f(z) = \dfrac{z+2}{z(z+3)^2}, f(z) \underset{z\to -3}{\longrightarrow} \infty, (z+3)f(z) = \dfrac{z+2}{z(z+3)} \underset{z\to -3}{\longrightarrow} \infty, (z+3)^2 \cdot f(z) = \dfrac{z+2}{z} \underset{z\to -3}{\longrightarrow} \dfrac{1}{3} \neq \infty, \Rightarrow$

$\Rightarrow z_0 = -3$ is a 2nd order pole for $f(z) \Rightarrow$ the residue in it will be:

$$\operatorname{Res}_{-3} f(z) = \lim_{z\to -3} \left((z+3)^2 \cdot f(z)\right)' = \lim_{z\to -3} \left(\frac{z+2}{z}\right)' = \lim_{z\to -3} \frac{z - z - 2}{z^2} = -\frac{2}{9}.$$

Answer: $-2/9$.