Problem:

Simplify the sum:

$$1-\frac{1}{5}+\frac{1}{4}-\frac{1}{25}+\frac{1}{16}-\frac{1}{125}+\frac{1}{64}-\cdots-\frac{1}{5^n}+\frac{1}{4^n}.$$

Ellipses and summation signs can't be used in the answer.

Solution:

Let's simplify the sum:

$$S=1-\frac{1}{5}+\frac{1}{4}-\frac{1}{25}+\frac{1}{16}-\frac{1}{125}+\frac{1}{64}-\cdots-\frac{1}{5^n}+\frac{1}{4^n}=1+\frac{1}{4}+\cdots+\frac{1}{4^n}-\left(\frac{1}{5}+\frac{1}{25}+\cdots+\frac{1}{5^n}\right),$$

Let's write the sum of geometric progressions:

$$1+\frac{1}{4}+\cdots+\frac{1}{4^n}=1\cdot\frac{1-\left(\frac{1}{4}\right)^{n+1}}{1-\frac{1}{4}}=\frac{4}{3}\cdot\left(1-\frac{1}{4^{n+1}}\right),$$

$$\frac{1}{5}+\frac{1}{25}+\cdots+\frac{1}{5^n}=\frac{1}{5}\cdot\frac{1-\left(\frac{1}{5}\right)^{n}}{1-\frac{1}{5}}=\frac{1}{4}\cdot\left(1-\frac{1}{5^n}\right)\Rightarrow S=\frac{4}{3}\cdot\left(1-\frac{1}{4^{n+1}}\right)-\frac{1}{4}\left(1-\frac{1}{5^n}\right)=$$

$$=\frac{13}{12}+\frac{1}{4\cdot5^n}-\frac{1}{3\cdot4^n}\Rightarrow S=\boxed{\frac{13}{12}+\frac{1}{4\cdot5^n}-\frac{1}{3\cdot4^n}}.$$

Answer: $\frac{13}{12}+\frac{1}{4\cdot5^n}-\frac{1}{3\cdot4^n}$.