Problem:

Let the set $A \subset \mathbb{R}$ is open, and $B \subset \mathbb{R}$ is closed. Prove that the set $A \setminus B$ is open, and $B \setminus A$ is closed.

Solution:

From the properties of operations on sets $\Rightarrow A \setminus B = A \cap \bar{B}$.
According to the definition of closed set, $B$ is closed $\Rightarrow \bar{B}$ is open, and the intersection of a finite number of open sets is open$\Rightarrow A \cap \bar{B}$ is open $\Rightarrow A \setminus B$ is open.
From the properties of operations on sets $\Rightarrow \overline{B \setminus A} = \overline{B \cap \bar{A}} = \bar{B} \cup \bar{\bar{A}} = \bar{B} \cup A, B$ is closed $\Rightarrow \bar{B}$ is open, $A$ is open $\Rightarrow \bar{B} \cup A$ is open $\Rightarrow \overline{B \setminus A}$ is open $\Rightarrow B \setminus A$ is closed, in accordance with the definition.