



Problem:

Using the theorem of nested intervals, prove that the segment is uncountable.

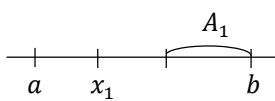
Solution:

Let's prove that the segment is uncountable, using the nested intervals theorem.

Let's prove that the segment $[a, b]$ is not countable.

Let's prove by contradiction. Let $[a, b]$ be countable $\Rightarrow$ is equivalent to $\mathbb{N} \Rightarrow$ its numbers can be enumerated: $x_1, x_2, x_3, \dots$

Let's divide $[a, b]$ into 3 equal parts. Let's select the part, which doesn't contain x_1 (on the boundary too). Let's denote the segment by A_1 .



Let's similarly divide A_1 into 3 equal parts and take the one, which doesn't contain x_2 , and so on. We obtain an infinite sequence of nested segments $A_1 \supset A_2 \supset \dots \supset A_n \supset \dots \Rightarrow$ according to nested intervals theorem $\Rightarrow \exists c \in A_n, n = 1, 2, \dots$ which means that $c \neq x_k, \forall k = 1, 2, 3, \dots$ but in that case $c \notin [a, b]$, we obtain a contradiction

$\Rightarrow [a, b]$ is uncountable.