Problem:

The binary relation P; find its domain and range. Check by the definition whether the relation P is reflexive, symmetric, antisymmetric, transitive. $P \subseteq Z^2, P = \{(x, y) \mid (x + 2 \cdot y)$ multiple of $2\}$.

Solution:

The domain of P:

$\delta_P = \{x \in Z \mid \exists y \in Z, xPy\}, \Rightarrow \delta_P = \{2k \mid k \in Z\}$, since $2y$ is divisible by 2 for all integer y.

The range of P:

$\rho_P = \{y \in Z \mid \exists x \in Z, xPy\}, \Rightarrow \rho_P = \mathrm{Z}, x + 2x = 3x$ is not divisible by 2 for odd $x \Rightarrow P$ is not reflexive.

We see that $(2,1) \in P$, but $(1,2) \notin P \Rightarrow P$ is not symmetric.

We see that $(2,4) \in P$, $(4,2) \in P$, but $2 \neq 4 \Rightarrow P$ is not antisymmetric.

From $(x, y) \in P, (y, z) \in P \Rightarrow (x + 2y)$ and $(y + 2z)$ are even $\Rightarrow x$ is even $\Rightarrow x + 2z$ will also be even $\Rightarrow (x, z) \in P \Rightarrow P$ is transitive.