Problem:

Prove the statement using mathematical induction:
$(8^{n} - 1)$ is a multiple of 7 for all natural $n \geq 1$.

Solution:

Let's prove the statement by induction on n.

1) Let $n = 1$, then $8^1 - 1 = 7$.

But 7 is divisible by 7, which means that when $n = 1$ the statement is true.

2) Let us prove that in this case for $(8^{k} - 1)$ is divisible by 7 without a remainder.

Let us prove that in this case, when $n = k + 1$ $(8^{k+1} - 1)$ is divisible by 7 without a remainder.

$8^{k+1} - 1 = 8 \cdot 8^{k} - 8 + 7 = 8 \cdot \left(8^{k} - 1\right) + 7$ is divisible by 7, i.e., according to the assumption the first term is divisible by 7. This means, by induction, the expression is a multiple of 7 for any natural number n.

The statement has been proven.