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Problem:

Prove that the sum of the squares of 3 consecutive integers can't be a perfect square.

Solution:

Let it be possible $\Rightarrow$ for some $n \in \mathbb{Z}$ we have:

$n^2 + (n+1)^2 + (n+2)^2 = k^2$, where $k \in \mathbb{Z} \Rightarrow n^2 + n^2 + 2n + 1 + n^2 + 4n + 4 = k^2 \Rightarrow 3(n^2 + 2n + 1) + 2 = = k^2$, it means k^2 leaves the remainder 2 when divided by 3, which is impossible, since when $k = 3m \Rightarrow k^2 \equiv \equiv 0(\mathrm{mod}\, 3)$, when $k = 3m + 1, k^2 = 9m^2 + 6m + 1 \equiv 1(\mathrm{mod}\, 3)$, when $k = 3m + 2 \Rightarrow k^2 = 9m^2 + 12m + 4 \equiv \equiv 1(\mathrm{mod}\, 3) \Rightarrow k^2$ can leave the remainder 0 or 1 when divided by 3 $\Rightarrow$ we have obtained a contradiction. It means the sum of 3 consecutive integers can't be a perfect square. The assertion is proved.