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Problem:

Prove the statement using mathematical induction:

$(n^3 + 11n) \vdots 6$, for all natural numbers $n \geq 1$.

Solution:

Let us prove it using the method of mathematical induction on n.

a) check the validity of the statement for $n = 1 \Rightarrow 1^3 + 11 \cdot 1 = 12 \vdots 6$, the statement is true ($a \vdots b$ means that a is divisible by b, where $a, b \in \mathbb{Z}$).

b) let the statement be true when $n = k$. Let us prove that it is also true when $n = k + 1$. We have $(k^3 + 11k) \vdots 6$, $\Rightarrow (k+1)^3 + 11(k+1) = k^3 + 11k + 3k^2 + 3k + 12$, but $(k^3 + 11k) \vdots 6, 12 \vdots 6, 3k^2 + 3k = 3k(k+1) \vdots 6$, since one of two consecutive natural numbers is even, $\Rightarrow$ $k(k+1) \vdots 2 \Rightarrow 3k(k+1) \vdots 6$ $\Rightarrow$ $\left((k+1)^3 + +11(k+1)\right) \vdots 6 \Rightarrow$ the statement is true for any natural number $n \geq 1$.