



Problem:

Calculate the indefinite inetrgal:

$$\int \frac{2x+3}{\sqrt[4]{x^2+3x+1}} dx.$$

Solution:

$$\int \frac{2x+3}{\sqrt[4]{x^2+3x+1}} dx = \int \left(\left(x + \frac{3}{2} \right)^2 - \frac{5}{4} \right)^{-\frac{1}{4}} \cdot (2x+3) dx = \boxed{x + \frac{3}{2} = y, \quad y^2 = z + \frac{5}{4} \Rightarrow}$$

$$= \int \left(y^2 - \frac{5}{4} \right)^{-\frac{1}{4}} \cdot 2y dy = \int \left(y^2 - \frac{5}{4} \right)^{-\frac{1}{4}} dy^2 = \int z^{-\frac{1}{4}} dz = \frac{4}{3} z^{-\frac{1}{4}+1} + c = \frac{4}{3} z^{\frac{3}{4}} + c =$$

$$= \frac{4}{3} \left(y^2 - \frac{5}{4} \right)^{\frac{3}{4}} + c = \frac{4}{3} \cdot \left(\left(x + \frac{3}{2} \right)^2 - \frac{5}{4} \right)^{\frac{3}{4}} + c = \frac{4}{3} (x^2 + 3x + 1)^{\frac{3}{4}} + c.$$

$$\text{Answer: } \frac{4}{3} (x^2 + 3x + 1)^{\frac{3}{4}} + c.$$