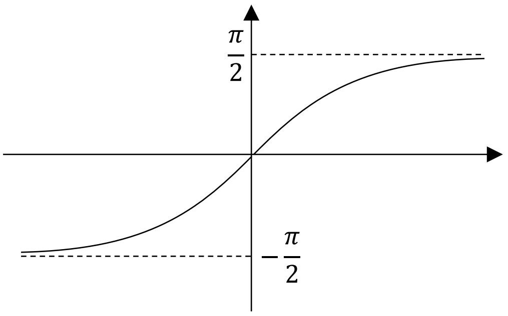
Problem:

Calculate the improper integral:

$$\int_{-\infty}^{+\infty} \frac{dx}{x^2+2x+2}.$$

Solution:

$$\int_{-\infty}^{+\infty} \frac{dx}{x^2+2x+2} = \int_{-\infty}^{0} \frac{dx}{(x+1)^2+1} + \int_{0}^{+\infty} \frac{dx}{(x+1)^2+1} = \lim_{A\to-\infty} \int_{A}^{0} \frac{dx}{(x+1)^2+1} + \lim_{B\to+\infty} \int_{0}^{B} \frac{dx}{(x+1)^2+1} =$$



$$= \lim_{A\to-\infty} \tan^{-1}(x+1)\Big|_{A}^{0} + \lim_{B\to+\infty} \tan^{-1}(x+1)\Big|_{0}^{B} =$$

$$\boxed{\int \frac{dx}{x^2+a^2} = \frac{1}{a}\tan^{-1}\frac{x}{a}}$$

$$= \tan^{-1} 1 - \lim_{A\to-\infty} \tan^{-1}(A+1) + \lim_{B\to+\infty} \tan^{-1}(B+1) - \tan^{-1} 1 =$$

$$= \frac{\pi}{2} + \frac{\pi}{2} = \pi, \quad \text{since} \quad \tan^{-1} x \xrightarrow[x\to+\infty]{} \frac{\pi}{2}, \quad \tan^{-1} x \xrightarrow[x\to-\infty]{} -\frac{\pi}{2}.$$

Answer: π.