



Problem:

Calculate the improper integral:

$$\int_{-\infty}^{+\infty} \frac{dx}{x^2 + 2x + 2}.$$

Solution:

$$\begin{aligned}\int_{-\infty}^{+\infty} \frac{dx}{x^2 + 2x + 2} &= \int_{-\infty}^0 \frac{dx}{(x+1)^2 + 1} + \int_0^{+\infty} \frac{dx}{(x+1)^2 + 1} = \lim_{A \rightarrow -\infty} \int_A^0 \frac{dx}{(x+1)^2 + 1} + \lim_{B \rightarrow +\infty} \int_0^B \frac{dx}{(x+1)^2 + 1} = \\ &= \lim_{A \rightarrow -\infty} \tan^{-1}(x+1) \Big|_A^0 + \lim_{B \rightarrow +\infty} \tan^{-1}(x+1) \Big|_0^B = \\ &\quad \text{Graph: A Cartesian coordinate system showing the curve } y = \tan^{-1}(x+1). \text{ The x-axis is horizontal and the y-axis is vertical. Dashed lines indicate the asymptotes at } y = \pm \frac{\pi}{2}. \text{ The curve passes through the origin and approaches the asymptotes as } x \rightarrow \pm \infty. \\ &\quad \boxed{\int \frac{dx}{x^2 + a^2} = \frac{1}{a} \tan^{-1} \frac{x}{a}} \\ &= \tan^{-1} 1 - \lim_{A \rightarrow -\infty} \tan^{-1}(A+1) + \lim_{B \rightarrow +\infty} \tan^{-1}(B+1) - \tan^{-1} 1 = \\ &= \frac{\pi}{2} + \frac{\pi}{2} = \pi, \text{ since } \tan^{-1} x \xrightarrow{x \rightarrow +\infty} \frac{\pi}{2}, \tan^{-1} x \xrightarrow{x \rightarrow -\infty} -\frac{\pi}{2}.\end{aligned}$$

Answer: π .