



Problem:

Examine the improper integral of the unbounded function for convergence:

$$\int_0^8 \frac{dx}{2x^3 + 5\sqrt{x}}.$$

Solution:

$$\int_0^8 \frac{dx}{2x^3 + 5\sqrt{x}} = \int_0^{2\sqrt{2}} \frac{2ydy}{2y^6 + 5y} = \int_0^{2\sqrt{2}} \frac{2dy}{2y^5 + 5}, \quad \boxed{\begin{matrix} x = y^2 \\ dx = 2ydy \end{matrix}} \Rightarrow \left| \frac{2}{2y^5 + 5} \right| \leq \frac{2}{5}, \quad \int_0^{2\sqrt{2}} \frac{2}{5} dy \text{ converges,}$$

which means the original integral converges, moreover, it absolutely converges.

Answer: the integral converges.