Problem:

Examine the improper integral for convergence:

$$\int_0^{+\infty} \frac{2xdx}{1+x^2}.$$

Solution:

$$\int_0^{+\infty} \frac{2xdx}{1+x^2} = \lim_{a\to+\infty} \int_0^a \frac{2xdx}{1+x^2} = \lim_{a\to+\infty} \int_0^a \frac{dx^2}{1+x^2} = \lim_{a\to+\infty} \ln(1+x^2)\Big|_0^a = \lim_{a\to+\infty} \ln(1+a^2) = +\infty, \Rightarrow$$

$\Rightarrow$ the integral diverges.

Answer: the integral diverges.