Problem:

Examine the improper integral for convergence:

$$\int_0^2 \frac{dx}{x}.$$

Solution:

$$\int_0^2 \frac{dx}{x} = \lim_{a\to 0+} \int_a^2 \frac{dx}{x} = \lim_{a\to 0+} \ln|x| \Big|_a^2 = \lim_{a\to 0+} (\ln 2 - \ln a) = \ln 2 - \lim_{a\to 0+} \ln a = +\infty \left(\ln a \underset{a\to 0+}{\longrightarrow} -\infty\right), \Rightarrow$$

$\Rightarrow$ the integral diverges.

Answer: the integral diverges.