



Problem:

Prove the equality:

$$\int_0^1 x^m (1-x)^n dx = \int_0^1 (1-x)^m x^n dx.$$

Solution:

$$\int_0^1 x^m (1-x)^n dx = \boxed{\begin{matrix} x = 1-y, \\ dy = -dx \Rightarrow \end{matrix}} = - \int_1^0 (1-y)^m y^n dy = \int_0^1 (1-y)^m y^n dy = \int_0^1 (1-x)^m x^n dx.$$