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Problem:

Find the value of the definite integral:

$$\int_{\ln 3}^{\ln 8} \frac{dx}{\sqrt{e^x+1}}.$$

Solution:

$$\int_{\ln 3}^{\ln 8} \frac{dx}{\sqrt{e^x+1}} = \int_2^3 \frac{1}{y}\cdot\frac{2y}{y^2-1}dy = 2\int_2^3 \frac{dy}{y^2-1} =$$

$\sqrt{e^x+1}=y, \quad e^x=y^2-1, \quad x=\ln(y^2-1), \quad dx=\frac{2y}{y^2-1}dy, \Rightarrow$
$x\in[\ln 3, \ln 8], \quad e^x\in[3,8], \quad y^2=e^x+1, \quad y\in[2,3]$

$$= \int_2^3 \frac{(y+1)-(y-1)}{y^2-1}dy = \int_2^3 \frac{dy}{y-1} - \int_2^3 \frac{dy}{y+1} = \ln|y-1|\Big|_2^3 - \ln|y+1|\Big|_2^3 =$$

$$= \ln 2 - \ln 1 - \ln 4 + \ln 3 = \ln 2 - 2\ln 2 + \ln 3 = \ln 3 - \ln 2 = \ln\frac{3}{2}.$$

Answer: $\ln(3/2)$.