Problem:

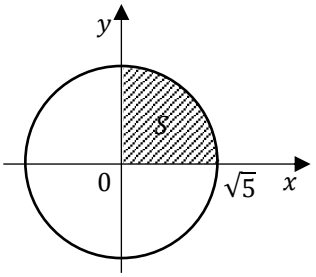
Find the value of the definite integral:

$$\int_0^{\sqrt{5}} \sqrt{15-3x^2}\,dx.$$

Solution:

This integral is equal to the area of the figure bounded by the straight lines $x=0, x=\sqrt{5}, y=0$ and the curve $y=\sqrt{15-3x^2}$.

$$y^2=15-3x^2, \qquad \frac{x^2}{5}+\frac{y^2}{15}=1,$$



this is an ellipse with semi-axes $a=\sqrt{5}, b=\sqrt{15}$, the area of the ellipse is πab

$$\Rightarrow \int_0^{\sqrt{5}} \sqrt{15-3x^2}\,dx = S = \frac{\pi ab}{4} = \frac{\sqrt{5}\sqrt{15}\pi}{4} = \frac{5\sqrt{3}}{4}\pi.$$

Answer: $\frac{5\sqrt{3}}{4}\pi$.