Problem:

Prove that the number $\sqrt{3}-\sqrt{2}$ is irrational.

Solution:

Let's prove from the contrary. Let $\sqrt{3}-\sqrt{2} \in \mathbb{Q}, \Rightarrow \sqrt{3}-\sqrt{2}=\frac{m}{n}$, where $m \in \mathbb{Z},\ n \in \mathbb{N}$, и $(m,n)=1$, $\Rightarrow$ $\left(\sqrt{3}--\sqrt{2}\right)^2 = m^2/n^2$, let's denote $a=m^2, b=n^2 \Rightarrow a \geq 0, b>0 \Rightarrow 3-2\sqrt{6}+2=a/b \Rightarrow$

$\Rightarrow \left(5-\frac{a}{b}\right)^2 = \left(2\sqrt{6}\right)^2 \Rightarrow (5b-a)^2 = 24b^2 \Rightarrow (5b-a)^2 \vdots b^2 \Rightarrow (5b-a) \vdots b \Rightarrow a \vdots b$, but $(m,n)=1 \Rightarrow$

$\Rightarrow (m^2,n^2)=1, i.e., (a,b)=1 \Rightarrow b=1 \Rightarrow (5-a)^2=24, |5-a|=\sqrt{24}$, but $\sqrt{24} \notin \mathbb{Q}$.

The assertion is proved.