Problem:

Examine the convergence of the series.

1) $\sum_{n=1}^{\infty} \frac{1}{2^n + 1}$ ,  2) $\sum_{n=2}^{\infty} \frac{2n}{\sqrt{n^2 - 1}}$.

Solution:

1) $$\sum_{n=1}^{\infty} \frac{1}{2^n + 1} < \sum_{n=1}^{\infty} \frac{1}{2^n}$$

The geometric series converges as a geometric progression with a denominator.

$q = \frac{1}{2} < 1 \Rightarrow$ according to the comparison test, the initial positive series also converges.

2) $$\frac{2n}{\sqrt{n^2 - 1}} = \frac{2}{\sqrt{1 - \frac{1}{n^2}}} \xrightarrow[n\to\infty]{} \frac{2}{1} \left(\frac{1}{n^2} \xrightarrow[n\to\infty]{} 0\right), \Rightarrow a_n \xrightarrow[n\to\infty]{} 2 \neq 0 \Rightarrow$$

$\Rightarrow$ the necessary condition for the convergence of the series is not satisfied $\Rightarrow$ the series diverges.

Answer: 1) converges, 2) diverges.