Problem:

Expand function $f(x) = e^{2x}$ into a power series in the vicinity of the point $x_0 = 3$.

Solution:

 $f(x) = e^{2x}$ let's find the expansion of f in the vicinity of the point $x_0 = 3$:

$$f(x) = f(x_0) + \frac{f'(x_0)}{1!}(x - x_0) + \dots + \frac{f^{(n)}(x_0)}{n!}(x - x_0)^n + \dots, \qquad f^{(n)}(x_0) = 2^n e^{2x_0} = 2^n e^6 \Rightarrow$$

$$\Rightarrow e^{2x} = e^6 + \frac{2e^6}{1!}(x-3) + \frac{2^2e^6}{2!}(x-3)^2 + \dots + \frac{2^ne^6}{n!}(x-3)^n + \dots = \sum_{n=0}^{\infty} \frac{2^ne^6}{n!}(x-3)^n,$$

We have obtained the expansion of f(x) in powers (x - 3).