Problem:

Calculate the integral:

$$\int_{-2\pi}^{2\pi} \sin^3 2x\sqrt{100-x^2}dx.$$

Solution:

Let's note that the integrand function $f(x) = \sin^3 2x\sqrt{1-x^2}$ is odd, since $f(-x) =$
$= \sin^3(-2x)\sqrt{1-(-x)^2} = (-\sin 2x)^3\sqrt{1-x^2} = -\sin^3 2x\sqrt{1-x^2} = -f(x) \Rightarrow f(-x) = -f(x).$

But for the odd function we have:

$$\int_{-2\pi}^{2\pi} f(x)dx = 0, \text{since } \int_{-2\pi}^{2\pi} f(x)dx = \int_{-2\pi}^{0} f(x)dx + \int_{0}^{2\pi} f(x)dx = \boxed{\text{replacement } y = -x} =$$

$$= \int_{-2\pi}^{0} f(-y)d(-y) + \int_{0}^{2\pi} f(x)dx = -\int_{0}^{2\pi} f(y)dy + \int_{0}^{2\pi} f(x)dx = 0 \Rightarrow \int_{-2\pi}^{2\pi} \sin^3 2x\sqrt{100-x^2}dx = 0.$$

Answer: 0.