Problem:

Find all values of the parameter a, for which the minimum value of the function $f(x) = 4x^2 - 4ax + a^2 - 2a +$ $+2$ on the segment $x \in [0; 2]$ is equal to 3.

Solution:

$f(x) = 4x^2 - 4ax + a^2 - 2a,\ \min\limits_{x\in[0;2]} f(x) = 3.\ f(x) = (2x - a)^2 - 2a + 2.$

The graph $y = f(x)$ is a parabola, the branches are directed up.

Let's consider 2 cases.

$The\ vertex\ of\ the\ parabola\ is\ the\ point\ x = \frac{a}{2}, y = f\left(\frac{a}{2}\right).$

а) $The\ vertex\ of\ the\ parabola\ on\ the\ segment\ [0; 2], i.e. \frac{a}{2} \in [0; 2] \Rightarrow \min\limits_{[0;2]} f(x) = f\left(\frac{a}{2}\right) = -2a + 2 = 3, \Rightarrow$

$\Rightarrow a = -\frac{1}{2}, \frac{a}{2} = -\frac{1}{4} \notin [0; 2], a\ contradiction.$

б) The vertex of the parabola outside the segment $[0; 2] \Rightarrow$ the minimum value $f(x)$ takes on one of the ends.

$[0; 2].\ f(0) = a^2 - 2a + 2, f(2) = a^2 - 10a + 18.$

$\frac{a}{2} \notin [0; 2] \Rightarrow a \notin [0; 4]. If\ a < 0 \Rightarrow f(2) > f(0)\ (a < 2) \Rightarrow f(0) = a^2 - 2a + 2 = 3, a^2 - 2a - 1 = 0,$

$a = 1 \pm \sqrt{2}, a < 0 \Rightarrow a = 1 - \sqrt{2}. If\ a > 4 \Rightarrow f(0) > f(2)\ (a > 2) \Rightarrow f(2) = a^2 - 10a + 18 = 3 \Rightarrow$

$\Rightarrow a^2 - 10a + 15 = 0, a = 5 \pm \sqrt{10}, a > 4 \Rightarrow a = 5 + \sqrt{10}.$

Answer: $a = 1 - \sqrt{2}$ и $a = 5 + \sqrt{10}$.