Problem:

Find the general solution of the linear homogeneous differential equation with constant coefficients:

$y''' + 5y'' - 6y' = 0.$

Solution:

$y''' + 5y'' - 6y' = 0$. The equation is homogeneous, linear, with constant coefficients $\Rightarrow$ let's write the characteristic equation:

$\lambda^3 + 5\lambda^2 - 6\lambda = 0 \Rightarrow \lambda(\lambda^2 + 5\lambda - 6) = 0, \quad \lambda(\lambda - 1)(\lambda + 6) = 0 \Rightarrow \lambda_1 = 0, \quad \lambda_2 = 1, \quad \lambda_3 = -6 \Rightarrow$

$\Rightarrow$ the general solution of the initial equation will be: $y = C_1 e^{\lambda_1 x} + C_2 e^{\lambda_2 x} + C_3 e^{\lambda_3 x} \Rightarrow$ $\boxed{y = C_1 + C_2 e^{x} + C_3 e^{-6x}}$, where C_1, C_2, C_3 are constant coefficients.