Problem:

Find the general solution of the linear homogeneous system of differential equations:

$$\begin{cases} \dot{x} = -y \\ \dot{y} = x \end{cases}.$$

Solution:

$\begin{cases} \dot{x} = -y \\ \dot{y} = x \end{cases} \Bigg| \Rightarrow$ let's differentiate the 1st equation $\Rightarrow \ddot{x} = -\dot{y} = -x \Rightarrow \ddot{x} + x = 0$, the characteristic equation will be: $\lambda^2 + 1 = 0 \Rightarrow \lambda_1 = i, \lambda_2 = -i \Rightarrow$ the general solution of the system will be:

$x = C_1 \cos t + C_2 \sin t, \Rightarrow y = -\dot{x} = -C_2 \cos t + C_1 \sin t.$

(when $\lambda_1 = i, \lambda_2 = -i$, the general solution of $\ddot{x} + x = 0$ will be $X = C_1 e^{it} + C_2 e^{-it}$, where C_1, C_2 are

arbitrary complex numbers, substituting $e^{it} = \cos t + i \sin t, e^{-it} = \cos t - i \sin t$, from the general solution let's separate the real part, which will have this form $C_1 \cos t + C_2 \sin t$).

Answer: $x = C_1 \cos t + C_2 \sin t, y = -C_2 \cos t + C_1 \sin t.$