



Problem:

Let's prove the relation  $X \times Y = A \times B \Leftrightarrow X = A, Y = B$ .

Solution:

a) Necessity: let's prove that  $X \times Y = A \times B \Rightarrow X = A, Y = B$ . Let's take any element from

$X \times Y: \forall x \in X, \forall y \in Y \Rightarrow (x, y) \in X \times Y = A \times B \Rightarrow (x, y) \in A \times B \Rightarrow x \in A, y \in B, \Rightarrow \forall x \in X, \forall y \in Y, \Rightarrow$   
 $\Rightarrow x \in A, y \in B \Rightarrow X \subseteq A, Y \subseteq B$ . Similarly, let's take  $\forall a \in A, \forall b \in B \Rightarrow (a, b) \in A \times B = X \times Y \Rightarrow$   
 $\Rightarrow (a, b) \in X \times Y \Rightarrow a \in X, b \in Y \Rightarrow \text{from } a \in A, b \in B \Rightarrow a \in X, b \in Y \Rightarrow A \subseteq X, B \subseteq Y$ .

We have obtained  $X \subseteq A \subseteq X, Y \subseteq B \subseteq Y \Rightarrow X = A, Y = B$ .

b) The sufficiency is obvious:  $X = A, Y = B \Rightarrow X \times Y = A \times B. \Rightarrow X \times Y = A \times B \Leftrightarrow X = A, Y = B$ .