Problem:

Solve the inequality, using the majorant method:

$\cos x - y^2 - \sqrt{y - x^2 - 1} \geq 0.$

Solution:

$\cos x - y^2 - \sqrt{y - x^2 - 1} \geq 0 \Rightarrow \sqrt{y - x^2 - 1} \leq \cos x - y^2, \qquad y - x^2 - 1 \geq 0 \Rightarrow y \geq x^2 + 1 \geq 1,$

on the other hand $\sqrt{y - x^2 - 1} \geq 0 \Rightarrow \cos x - y^2 \geq 0 \Rightarrow \cos x \geq y^2 \geq 1\ (y \geq 1), but\ \cos x \leq 1 \Rightarrow$

$\Rightarrow 1 \geq \cos x \geq y^2 \geq 1 \Rightarrow 1 = y^2 = \cos x, y \geq 1 \Rightarrow y = 1, \cos x = 1 \Rightarrow x = 2\pi k, k \in \mathbb{Z},$

$but\ y \geq x^2 + 1 \Rightarrow 1 \geq x^2 + 1 \Rightarrow x^2 \leq 0 \Rightarrow x = 0 \Rightarrow$ We obtained the unique solution to the inequality:

$x = 0, y = 1.$

$Answer:\ x = 0, y = 1.$