Problem:

Solve the inequality, using the majorant method:

$-|y| + x - \sqrt{x^2 + y^2 - 1} \geq 1.$

Solution:

$-|y| + x - \sqrt{x^2 + y^2 - 1} \geq 1 \Rightarrow (*)\ x - 1 - |y| \geq \sqrt{x^2 + y^2 - 1} \geq 0 \Rightarrow x - 1 - |y| \geq 0 \Rightarrow$

$\Rightarrow x \geq 1 + |y|\ (**) \Rightarrow \textit{in this case } x^2 + y^2 \geq x^2 \geq 1.$

Now let's square both parts of $(*)$ (both parts are non-negative) $\Rightarrow$

$\Rightarrow x^2 + 1 + y^2 - 2x + 2|y| - 2x|y| \geq x^2 + y^2 - 1 \Rightarrow 1 \geq x - |y| + x|y|, \Rightarrow x - 1 - |y| \leq -x|y|,$

$\textit{but from } (**) \textit{ we have } x - 1 - |y| \geq 0, \textit{where } x \geq 1 \Rightarrow\ -x|y| \geq x - 1 - |y| \geq 0 \Rightarrow x|y| \leq 0, x \geq 1 \Rightarrow$

$\Rightarrow |y| \leq 0 \Rightarrow y = 0 \Rightarrow x \geq 1, \textit{and evidently in this case }\ x - \sqrt{x^2 - 1} \geq 1, \textit{as long as } (x-1)^2 \geq x^2 - 1.$

So, the solution to the inequality will be the points (x, y): $\begin{cases} y = 0 \\ x \geq 1 \end{cases}.$

Answer: $\{\,(x, 0) \mid x \geq 1\,\}$.